Amendments to the Specification:

After line 7, page 11 of the as-filed application, please insert the following paragraphs:

For example, and as discussed in this reference, multi-variable polynomials have been successfully employed to realize functional index fits. It is well known that the quality of the interpolation does not increase as the order of the approximation increases. C. Runge examined a famous example in one dimension, on the interval -5<x<5 where he showed that the maximum of the error approached infinity as the polynomial order increased.

To avoid such problems, the domain may be partitioned into smaller sub-intervals. A piecewise polynomial interpolation is then obtained over these smaller intervals. Perhaps the most popular is a cubic; both the first and second derivatives are continuously differentiable. When the approximating polynomial g(x) of the original function f(x) satisfies the property that g=f and g'=f ' at the end points of the subinterval, then

$$\int_a^b f''(x)^2\,dx \geq \int_a^b g''(x)^2\,dx. \qquad (19)$$

This simply infers that the "strain" energy minimized by the spline approximates well the energy of the original function. That fact along with the ease of computing the partial derivatives in any one direction make a multivariable spline an ideal choice for an approximating function.

For example, consider a function fit to only one variable. The truncated Taylor series

$$f(x)=\sum_{i=0}^{n}\frac{(x-a)^i}{i!}D^i f(a), \qquad (20)$$

provides a satisfactory approximation to f(x) provided that x is sufficiently smooth and f is sufficiently close to a. If the interval is large, requiring large values of n, spurious oscillatory problems may result. This of course precipitated the use of splines in which the interval was truncated and fit with polynomials of lower degree each with several continuous derivatives to generate smooth functions. The pp-form of a spline provides a description of the functional

in terms of intermediate breakpoints (or knots) $\xi_{1,\ldots,}\xi_{m,}$ and coefficients c_{ij} to be determined for a spline of order k so that

$$f(x)=\sum_{j=1}^{m}\sum_{i=1}^{k}\frac{(x-\xi_j)^{k-i}}{(k-i)!}c_{ij} \tag{21}$$

For this analysis, the B-form of the spline is used where

$$f(x)=\sum_{j=l}^{n}a_{ij}B_{j,k}(x). \tag{22}$$

where $B_{j,k}$ is the j^{th} B-spline of order k with knot sequence $t_1,t_{2,\ldots,}t_{n+k}$. The B form has become the standard way to represent a spline during its construction. The knots refer simply to the sub-intervals. If f was related to 3 variables x, y, and z, the trivariate tensor spline approximation would be

$$f(x,y,z)=\sum_q\sum_r\sum_p c(q,r,p)B_{r,k}(x)B_{q,k}(y)B_{p,k}(z). \tag{23}$$

Note that as additional variables are incorporated, they are handled as a product of the polynomials used to work with one dimension. The task is to determine the trivariate coefficients c. For this purpose consider f defined as $\sin(x)*(\cos(y)*\tan(z))^2$. The variables x, y, and z are allowed to vary from 0 to $2\pi/5$ each with 20 intermediate points in a nested do loop. The fit of these 8,000 points with only 729 coefficients has a max error of 0.0291. This function is smoothly differentiable.

After line 12, page 11 of the as-filed application, please insert the following paragraphs:

For example, and as described in this reference, a performance objective may be to choose b (thickness of a backiron) and t (thickness of a magnet) to maximize the average magnetic field in a 4" air gap divided by the weight of a magnetic vehicle. The magnets have, for example, a residual B field of 1.25T and a working coercivity of 955200 A/m. The carbon steel backiron thickness b may range from 0.5 - 10 inches in thickness, in a total of ten steps. The magnet thickness t may be stepped through the range of 1 - 12 inches with a step size of one inch. The total number of test cases to cover the range of values dictated by both of these quantities changing at the same time is 120.

With an initial backiron thickness of 0.5 inches, everything above the interfacial line separating the magnet and the iron may be scaled vertically from a range of 1 – 20. Everything below the interfacial line between the magnets and the iron may be scaled from a range of 1 - 12. To maintain an air gap of 4", the mid point of the air gap may also be scaled in step with the magnet to maintain the separation distance of 2". Thus, there are a total of three parametric scaling loops for this analysis, one for the backiron, one for the magnets, and one to adjust the horizontal Neumann Boudnard representing the middle of the air gap.

We seek the solution of the vector potential due to source currents using fictitious surface currents placed on the interface separating dissimilar materials. The equation dictating the magnetic field involves a single integral equation for the unknown fictitious surface current,

$$\mu_o \int_{s'} \frac{\partial G(r,r')}{\partial n} \vec{K}_f dS' + \frac{1+\mu}{1-\mu} \frac{\mu_o \vec{K}_f(r)}{2} = B_t(s) \qquad (24)$$

where $\mu = \frac{\mu_1}{\mu_2}$ *and* $\vec{B}_t$ is the tangential B field due to all other external magnetic sources.

Equation (24) must be altered slightly to account for media saturation. Additional volumetric sources equal to $\nabla \times \vec{M}$ must in general be distributed throughout the volume of

the iron. Their magnitude is smaller than the surface currents and is indeed zero when the media is linear. The general boundary element equation used for saturable media is

$$\vec{B}=\vec{B}_o+\oint_{\Gamma}\nabla\times(K(r')G(r,r'))\,ds'+\int_{\gamma}V\times(J_m(r')G(r,r'))\,dV \tag{25}$$

Obviously these volume currents, $\vec{J}_m=\nabla\times\vec{M}$, are unknown a priori. Solution of the problem is as follows. First, assume $J_m=0$; second, compute the B field everywhere; third, check all saturable regions to determine whether $\nabla\times\vec{M}\neq 0$; and substitute $\vec{J}_m=\nabla\times\vec{M}$. The process is then repeated until no change in these magnetization currents is witnessed. Under relaxation can be utilized to accelerate conversion, with an under relaxation parameter of about 0.3. Throughout this process the magnetization current is always a source term and is treated as known. It is computed after the fact and forms a contribution to the right hand side of the equations.

Once computed, the two dimensional variation of median air gap field with magnet thickness and backiron thickness must be accurately modeled. A fairly good solution was found by first considering the case where the backiron thickness was equal to 0. The working magnetic field in the air gap for this case can be modeled quite accurately using a quadratic polynomial fit as

$$B_{avg}=\sum_{i=1}^{3}c_i t^{3-i}=c_1t^2+c_2t+c_3 \tag{26}$$

Now the issue arises as to how the introduction of magnet backiron begins to complicate the determination of the working B field. Of course, the expected magnetic field in the gap will increase with backiron thickness. However, the advantage gained by adding backiron is somewhat inversely proportional to the magnet thickness itself, since with very thick magnets, the iron begins to saturate and becomes less effective for carrying flux. It was thus reasoned that any adaptation of equation (26) must be realized using a parameter which reflects this effect, one such parameter being simply the backiron thickness "t" divided by the magnet thickness "b". A secondary polynomial fit was performed in an attempt to account for the enhancement in the magnetic field expected by the addition of backiron as a multiplier on the formula found when no backiron exists i.e.,

$$B_{avg} = \left(\sum_{i=1}^{3} c_{\,i} t^{3-i}\right)\left(c_4\left(\frac{b}{t}\right)^2 + c_5\left(\frac{b}{t}\right) + c_6\right). \tag{27}$$

A least square polynomial analysis was used to determine the parameters as

$$B_{avg} = \left(\sum_{i=1}^{3} c_i t^{3-i}\right)\left(c_4\left(\frac{b}{t}\right)^2 + c_5\left(\frac{b}{t}\right) + c_6\right)$$

$$= \left(\sum_{i=1}^{3} c_i t^{i-1}\right)\left(-0.185\left(\frac{b}{t}\right)^2 + 1.682\left(\frac{b}{t}\right) + 0.92\right) \tag{28}$$

The polynomial match is usually good to less than 5%.